2022 AB/BC #4
(no calculator)

(a)

$$r''(8.5) \approx \frac{r'(10)-r'(7)}{10-7} = \boxed{\frac{-3.8-(-4.4)}{10-7}\frac{\frac{cm}{day}}{day}} \text{ or } 0.2\frac{cm}{day^2}$$

(b)

Since r is twice differentiable, then r' is differentiable and, hence, r' is also continuous.

So, by the Intermediate Value Theorem, r' takes on all values between $r'(0)=-6.1$ and $r'(3)=-5.0$.

Hence, since $-6.1=r'(0)<-6<-5=r'(3)$, there is a time on $[0,3]$ for which $r'(t)=-6$.

(c)

Right Riemann sum with 4 subintervals in the table:

$$\int_0^{12} r'(t)dt \approx (3-0)r'(3)+(7-3)r'(7)+(10-7)r'(10)+(12-10)r'(12)$$

$$= \boxed{3(-5)+4(-4.4)+3(-3.8)+2(-3.5)} \text{ or } -51\text{cm}$$

(d)

$\frac{dh}{dt}=-2$, and when $t=3$, then $r=100$ and $h=50$. Find $\frac{dV}{dt}$ when $t=3$.

$$V=\frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt}=\frac{1}{3}\pi\left(r^2\frac{dh}{dt}+h2r\frac{dr}{dt}\right)$$ using the product rule. Note: $\left.\frac{dr}{dt}\right|_{t=3}=-5$ from the table

$$\left.\frac{dV}{dt}\right|_{t=3} = \boxed{\frac{1}{3}\pi\left((100)^2(-2)+(50)(2)(100)(-5)\right)} \text{ or } -\frac{70000\pi}{3}$$

Note:

We know that the radius and height are not proportional in this case because dh/dt is constant, and dr/dt varies depending on the value of t, as given in the table. The only way the dimensions are proportional is if the shapes are similar, which would be true if the cone was facing with the point end down, but in this case the circular base is at the bottom, so while the shape continues to be a cone, it is not similar to the original cone, so the dimensions are not proportional. Thus the product rule is the only way to solve the problem.