2022 AB/BC #1
(calculator-active)

(a)

$A(t)=450\sqrt{\sin(0.62t)}$ is the rate at which vehicles arrive at the toll plaza in vehicles per hour from 5am until 10am. The number of vehicles that arrive at the toll plaza from 6am ($t=1$) until 10am ($t=5$) is $\boxed{\int_1^5 A(t)\,dt}$.

(b)

The average value of the rate at which the vehicles arrive at the toll plaza from time $t=1$ to time $t=5$ is $\frac{1}{5-1}\int_1^5 A(t)dt = \boxed{375.5369662}$ or 375.536 or 375.537 $\frac{vehicles}{hour}$

(c)

$A'(1)=148.9472908>0$

The rate at which the vehicles arrive at the toll plaza at 6am is $\boxed{\text{increasing}}$ because $A'(t)>0$ when $t=1$.

(d)

When $A(t)\geq 400$, $N(t)=\int_a^t \left(A(t)-400\right)dt$ for $a\leq t\leq 4$.

We want the absolute maximum value of $N(t)$ for $a\leq t\leq 4$.

This will occur when $t=a$, or when $t=4$, or when $N'(t)=0$.

$N'(t)=A(t)-400=0 \Rightarrow A(t)=400 \Rightarrow t=1.4693716=t_1$ and $t=3.5977133=t_2$

(we knew t_1 was going to be a since we know the line started forming at $t=a$)

Comparing the values of N at the candidates:

$N(a)=\int_a^a \left(A(t)-400\right)dt=0$

$N(t_1)=\int_a^{t_1} \left(A(t)-400\right)dt=0$

$N(t_2)=\int_a^{t_2} \left(A(t)-400\right)dt=71.254$

$N(4)=\int_a^4 \left(A(t)-400\right)dt=62.338$

So the greatest number of vehicles in line at the toll plaza in the time interval $a\leq t\leq 4$ is $\boxed{71}$ and this occurs at time $t=3.5977133$.