2023 BC #2
(calculator active)

(a)

$\frac{dx}{dt} = x'(t) = e^{\cos t}$ and $y = 2\sin t$ so $y'(t) = 2\cos t$ and the velocity vector is $\left\langle e^{\cos t}, 2\cos t \right\rangle$

$x''(t) = -\sin t \bullet e^{\cos t}$ and $y''(t) = -2\sin t$ so the acceleration vector is $\left\langle -\sin t \bullet e^{\cos t}, -2\sin t \right\rangle$

The acceleration vector when $t = 1$ is $\left\langle -\sin 1 \bullet e^{\cos 1}, -2\sin 1 \right\rangle = \boxed{\left\langle -1.44440611, -1.6829419 \right\rangle}$

or $\left\langle -1.444, -1.683 \right\rangle$ or $\left\langle -1.444, -1.682 \right\rangle$

(b)

Speed = $\sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2} = 1.5$

Solve this on your calculator for the interval $[0, \pi]$.

The first time speed is 1.5 on this interval is when $\boxed{t = 1.2544723}$ or $t = 1.254$

(c)

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{\left.\frac{dy}{dt}\right|_{t=1}}{\left.\frac{dx}{dt}\right|_{t=1}} = \frac{2\cos 1}{e^{\cos 1}} = \boxed{0.629531096} \text{ or } 0.630 \text{ or } 0.629$$

$$x(1) = x(0) + \int_0^1 x'(t)\,dt = 1 + \int_0^1 e^{\cos t}\,dt = \boxed{3.341574842} \text{ or } 3.342 \text{ or } 3.341$$

(d)

Total distance on $[0, \pi]$ $= \int_0^{\pi} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt = \boxed{6.034611337}$ or 6.035 or 6.034