2023 AB/BC #4
(no calculator)

(a)

$f'(6)=0$, but $f'(x)>0$ on $(2,6)$ and $f'(x)>0$ on $(6,8)$.

So $f'(x)$ does not change signs at $x=6$.

Hence, f has $\boxed{\text{neither}}$ a relative minimum nor a relative maximum at $x=6$.

(b)

f is concave down when f' is decreasing and this occurs on the intervals $\boxed{(-2,0) \text{ and } (4,6)}$.

(c)

$\lim\limits_{x\to 2}\dfrac{6f(x)-3x}{x^2-5x+6} \quad \to \quad \lim\limits_{x\to 2}\left(6f(x)-3x\right)=6f(2)-3(2)=6(1)-6=0$

Note: $f(x)$ is differentiable so it is continuous at x=2. So $\lim\limits_{x\to 2} f(x) = f(2) = 1$.

$\lim\limits_{x\to 2}\left(x^2-5x+6\right)=4-10+6=0$

This makes the limit above the indeterminate form $\dfrac{0}{0}$ so, applying L'Hospitals Rule:

$$\lim_{x\to 2}\frac{6f(x)-3x}{x^2-5x+6}=\lim_{x\to 2}\frac{6f'(x)-3}{2x-5}=\frac{6f'(2)-3}{2(2)-5}=\boxed{\frac{6(0)-3}{2(2)-5}} \text{ or } 3$$

(d)

The candidates for the absolute minimum value of f on $[-2,8]$ are the endpoints of the interval, $x=-2$ and $x=8$ or where $f'(x)=0$ and this occurs when $x=-1$, $x=2$, and $x=6$.

$$f(-2)=f(2)-\int_{-2}^{2} f'(x)\,dx=1-\left[\frac{1}{2}(1)(2)-\frac{1}{2}(1)(2)-\frac{1}{2}(2)(2)\right]=3$$

$$f(-1)=f(2)-\int_{-1}^{2} f'(x)\,dx=1-\left[-\frac{1}{2}(3)(2)\right]=1+3=4$$

$f(2)=1$

$f(6)$ is neither a maximum or minimum from part (a) and not an endpoint. (*But it equals* $7-\pi$)

$$f(8)=f(2)+\int_{2}^{8} f'(x)\,dx=1+\frac{1}{2}(2)(2)+4(2)-\frac{1}{2}\pi(2)^2=11-2\pi$$

So the absolute minimum value of f is $\boxed{1}$ and it occurs at $x=2$.