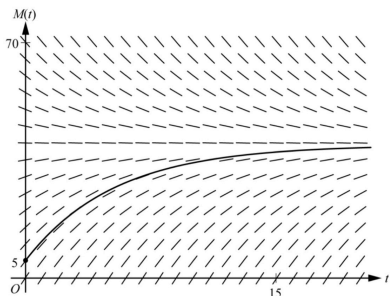


2023 AB/BC #3
(no calculator)

(a)



(b)

$$M'(t) = \frac{dM}{dt} = \frac{1}{4}(40 - M), \quad M(0) = 5$$

$$\text{tangent line at } t = 0: M - 5 = M'(0)(t - 0) \rightarrow M = 5 + M'(0)t$$

$$M'(0) = \frac{1}{4}(40 - 5) = \frac{35}{4}. \quad \text{So tangent line at } t = 0: M = 5 + \frac{35}{4}t$$

$$\text{Using this tangent line: } M(2) \approx \boxed{5 + \frac{35}{4}(2)^\circ\text{C}} \text{ or } \frac{45}{2}^\circ\text{C}$$

(c)

$$\frac{d^2M}{dt^2} = \frac{1}{4} \left(0 - \frac{dM}{dt} \right) = -\frac{1}{4} \left(\frac{dM}{dt} \right) = \boxed{-\frac{1}{4} \left(\frac{1}{4}(40 - M) \right)} = -\frac{1}{16}(40 - M)$$

$$\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M) < 0 \text{ for all } t \text{ since } M < 40 \text{ for all } t.$$

Hence M is concave down for $(0, 2]$ so the approximation for $M(2)$ in part (b)

is an overestimate for the actual value of $M(2)$.

(d)

$$\frac{dM}{dt} = \frac{1}{4}(40 - M)$$

$$\frac{dM}{40 - M} = \frac{1}{4} dt$$

$$\int \frac{dM}{40 - M} = \int \frac{1}{4} dt$$

$$-\ln|40 - M| = \frac{1}{4}t + C \quad \rightarrow \quad M(0) = 5, \text{ so } -\ln|40 - 5| = \frac{1}{4}(0) + C \rightarrow C = -\ln 35$$

$$\text{and with the given condition that } M = 5, 45 - M > 0, \text{ so } |40 - M| = 40 - M.$$

$$-\ln(40 - M) = \frac{1}{4}t - \ln 35$$

$$\ln(40 - M) = -\frac{1}{4}t + \ln 35$$

$$40 - M = e^{-\frac{1}{4}t + \ln 35} \quad \rightarrow$$

$$\boxed{M = 40 - e^{-\frac{1}{4}t + \ln 35}}$$

or an equivalent expression