2023 AB #5
(no calculator)

(a)

$h(x) = f\big(g(x)\big)$

$h'(x) = f'\big(g(x)\big)g'(x)$

Using values from the given chart:

$h'(7) = f'\big(g(7)\big)g'(7) = f'(0) \bullet 8 = \boxed{\frac{3}{2} \bullet 8}$ or 12

(b)

$k'(x) = \big(f(x)\big)^2 g(x)$

$k''(x) = \big(f(x)\big)^2 g'(x) + g(x) \bullet 2f(x)f'(x)$

$k''(4) = \big(f(4)\big)^2 g'(4) + g(4) \bullet 2f(4)f'(4)$

$= \big(4\big)^2 \bullet 2 + (-3) \bullet 2 \bullet 4 \bullet 3 = 32 - 72 = -40 < 0$

k is $\boxed{\text{concave down}}$ at the point where $x = 4$ since $k''(4) < 0$.

(c)

$m(x) = 5x^3 + \int_0^x f'(t)\,dt$

$m(2) = 5(2)^3 + \int_0^2 f'(t)\,dt$

$= 5(2)^3 + f(2) - f(0)$ (using the Fundamental Theorem of Calculus)

$= \boxed{5(2)^3 + 7 - 10}$ or 37

(d)

$m'(x) = 15x^2 + f'(x)$ (using the Fundamental Theorem of Calculus)

$m'(2) = 15(2)^2 + f'(2) = 60 + (-8) = 52 > 0$

m is $\boxed{\text{increasing}}$ at $x = 2$ since $m'(2) > 0$.