2019 AB #6
(no calculator)

(a)

$y = 4 + \frac{2}{3}(x-2) = 4 + \frac{2}{3}x - \frac{4}{3} = \frac{2}{3}x + \frac{8}{3}$

$y'(x) = \frac{2}{3}$. So $h'(2) = \boxed{\frac{2}{3}}$ This is the slope of the line tangent to h at $x = 2$.

(b)

$a(x) = 3x^3 h(x)$

$a'(x) = 3x^3 h'(x) + 9x^2 h(x)$

$a'(2) = 3(2^3)h'(2) + 9(2^2)h(2) = \boxed{3(2^3)\left(\frac{2}{3}\right) + 9(2^2)(4)}$ 160

(c)

$h(x) = \frac{x^2 - 4}{1 - (f(x))^3}$ and f, f', and h are continuous since they are differentiable,

so $\lim_{x\to 2} h(x) = h(2)$ and $\lim_{x\to 2} f(x) = f(2)$ and $\lim_{x\to 2} f'(x) = f'(2)$.

Since L'Hospital's Rule applies, then

$\lim_{x\to 2}\left(1 - (f(x))^3\right) = 0 \Rightarrow 1 - (f(2))^3 = 0 \Rightarrow (f(2))^3 = 1 \Rightarrow \boxed{f(2) = 1}$

also, since L'Hospital's rule applies,

$$h(2) = \lim_{x\to 2}\left(\frac{x^2-4}{1-(f(x))^3}\right) = \lim_{x\to 2}\left(\frac{2x}{-3(f(x))^2 f'(x)}\right) = \frac{4}{-3(f(2))^2 f'(2)} = \frac{4}{-3(1)^2 f'(2)} = 4$$

So, $\boxed{f'(2) = -\frac{1}{3}}$

(d)

We know that both g and h are differentiable so they are both continuous.

So $\lim_{x\to 2} g(x) = g(2) = 4$ and $\lim_{x\to 2} h(x) = h(2) = 4$.

Since $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, then

$g(2) \le k(2) \le h(2) \Rightarrow 4 \le k(2) \le 4$. So $k(2) = 4$

also

$\lim_{x\to 2} g(x) \le \lim_{x\to 2} k(x) \le \lim_{x\to 2} h(x) \Rightarrow 4 \le \lim_{x\to 2} k(x) \le 4$. So by the Squeeze Theorem $\lim_{x\to 2} k(x) = 4$.

Since $k(2) = \lim_{x\to 2} k(x)$, then k is continuous at $x = 2$.