2019 AB #5
(no calculator)

(a)

The top curve is $h(x)$ and the bottom curve is $g(x)$.

Area of $R = \int_0^2 \left(h(x) - g(x)\right)dx$

$$= \left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\Bigg|_0^2$$

$$= 8x - \frac{2}{3}(x-1)^3 - \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\Bigg|_0^2$$

$$= \boxed{\left(8(2) - \frac{2}{3}(2-1)^3 - \frac{6}{\pi}\sin\left(\frac{\pi}{2}(2)\right)\right) - \left(8(0) - \frac{2}{3}(0-1)^3 - \frac{6}{\pi}\sin\left(\frac{\pi}{2}(0)\right)\right)} \quad \frac{44}{3}$$

(b)

Volume $= \int_0^2 A(x)\,dx = \ln\left(x+3\right)\Big]_0^2 = \boxed{\ln 5 - \ln 3} \quad \ln\frac{5}{3}$

Note: Did not need $|x+3|$ since $x+3>0$ on the interval.

(c)

Axis of rotation is $y = 6$, so "big" radius is $6 - g(x)$ and "little" radius is $6 - h(x)$.

Volume $= \boxed{\pi\int_0^2 \left[\left(6 - g(x)\right)^2 - \left(6 - h(x)\right)^2\right]dx}$